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THE RELATION BETWEEN PLUMB-LINE DEFLECTION,
GRADIENTS, AND RADII OF CURVATURE OF A GEOID

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Summary: A method is given for deducing the formulas for studying the geoidal surface with the aid of gradients of the terrestrial gravitational field.

I

$$\xi_x = -\frac{I_x}{J}, \quad \eta = -\frac{I_y}{J} \quad (1)$$

With the aid of these two equations, we shall find the analytical relation between: the component plumb-line deflections ξ and η , the second derivatives $T_{xx} = \frac{d^2T}{dx^2}$, $T_{yy} = \frac{d^2T}{dy^2}$ and $T_{xy} = \frac{d^2T}{dxdy}$, and the radius of curvature K of the level surface of a geoid:

$$W(x, y, z) = C$$

- 2 -

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Differentiating (1), respectively with respect to x and y , we obtain

$$\begin{aligned} d\xi &= -\frac{1}{g} [\bar{T}_{xx} + T_{xy} \operatorname{tg} A] dx, \\ d\eta &= -\frac{1}{g} [\bar{T}_{yy} + T_{xy} \operatorname{ctg} A] dy, \end{aligned} \quad (2)$$

where A is the azimuth of the element of length $dl = \sqrt{dx^2 + dy^2} = \frac{dx}{\cos A} = \frac{dy}{\sin A}$

If the first equation of (2) is multiplied by $\sin A$ and the second equation by $\cos A$, then after subtracting one equation from the other we obtain:

$$d\xi \sin A - d\eta \cos A = T_{xy}^{(0)} dl, \quad (3)$$

where

$$T_{xy}^{(0)} = \frac{1}{2} T_A \sin 2A + T_{xy} \cos 2A, \quad (4)$$

hence, after integration, we obtain Eötös's equation:

$$(\xi_2 - \xi_1) \sin A - (\eta_2 - \eta_1) \cos A = \frac{1}{g} \int_{\xi_1}^{\xi_2} T_{xy}^{(0)} dl, \quad (5)$$

which finds application in the study of geoidal surfaces.

III

After the equations of (2) are multiplied respectively by $\cos A$ and $\sin A$, we add them term by term. As a result of a not inconsiderable number of transformations, we obtain:

$$\begin{aligned} d\xi \cos A + d\eta \sin A &= \\ &= -\frac{1}{g} [\bar{T}_{xx} \cos^2 A + \bar{T}_{yy} \sin^2 A + 2T_{xy} \sin A \cos A] dl, \end{aligned} \quad (6)$$

hence, after integrating in the same interval as used previously, we obtain:

$$\begin{aligned} (\xi_2 - \xi_1) \cos A + (\eta_2 - \eta_1) \sin A &= \\ &= -\frac{1}{g} \int_{\xi_1}^{\xi_2} [\bar{T}_{xx} \cos^2 A + \bar{T}_{yy} \sin^2 A + 2T_{xy} \sin A \cos A] dl. \end{aligned} \quad (7)$$

The equation just obtained above may be employed to determine the radius of curvature of a level surface of a geoid. Thus, it is sufficient to turn our attention to the nature of the integrand in function (7). It represents the negative quantity of the product of surface curvature $(-\frac{1}{R})$ multiplied by the

acceleration of the force of gravity (g), that is to say it equals (2):

$$-\frac{g}{R} = \bar{T}_{xx} \cos^2 A + \bar{T}_{yy} \sin^2 A + 2T_{xy} \sin A \cos A. \quad (8)$$

Consequently, instead of (6), we have:

$$\frac{d\xi \cos A + d\eta \sin A}{dl} = \frac{1}{R}, \quad (9)$$

which gives, in the planes of the meridian and of the primary vertical (normal), the equalities:

$$\frac{d\xi}{dl} = \frac{1}{R_M}, \quad \frac{d\eta}{dl} = \frac{1}{R_N}. \quad (10)$$

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We note, for clarity, that $R_L \neq$ and $R_H \neq N \sqrt{2}$. The expression (7) determines the deviation of the curvature of the geoid from the curvature of the surface of reference (deviation). Therefore, in order to obtain the complete curvature of a geoid, it is obviously necessary to calculate also the curvature of the surface, relative to which the geoidal surface is studied.

IV

We return again to the equations in (2) and we set $A = \alpha + (A - \alpha)$, where α is the azimuth of the vector of plumb-line deflection $\varepsilon = \sqrt{\xi^2 + \eta^2}$. Hence, for the case $A - \alpha = 0$, we obtain:

$$\begin{aligned} d\xi &= -\frac{1}{g} [T_{xx} + T_{xy} \operatorname{tg} \alpha] dx, \\ d\eta &= -\frac{1}{g} [T_{yy} + T_{xy} \operatorname{ctg} \alpha] dy, \end{aligned} \quad (11)$$

Hence, keeping in mind that:

$$\begin{aligned} d\xi &= d\varepsilon \cdot \cos \alpha, & dx &= d\varepsilon \cdot \cos \alpha, \\ d\eta &= d\varepsilon \cdot \sin \alpha, & dy &= d\varepsilon \cdot \sin \alpha, \end{aligned}$$

we obtain:

$$T_{xx} + T_{xy} \operatorname{tg} \alpha = T_{yy} + T_{xy} \operatorname{ctg} \alpha. \quad (12)$$

Consequently,

$$\operatorname{tg} \alpha - \operatorname{ctg} \alpha = \frac{T_{\Delta}}{T_{xy}}, \quad \frac{d\xi}{d\eta} = \operatorname{ctg} \alpha. \quad (13)$$

Setting:

$$\operatorname{tg} \alpha - \operatorname{ctg} \alpha = -\frac{2}{\operatorname{tg} 2\alpha}, \quad (14)$$

we obtain:

$$\operatorname{tg} 2\alpha = -\frac{2T_{xy}}{T_{\Delta}}. \quad (15)$$

Equation (15) determines α in the direction of the plumb-line deflection ε if T_{Δ} and T_{xy} are known.

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- 4 -

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